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EXAMPLES OF NON-APPLICABLE SURFACES HAVING THE SAME GAUSSIAN CURVATURE AT CORRESPONDING POINTS

BY ARTHUR SULLIVAN GALE

Stäckel and Wangerin* have given examples of pairs of non-applicable surfaces which have the same total curvature at corresponding points; *i. e.*, examples showing that the necessary condition for applicability afforded by Gauss's theorem is not sufficient in general, although it is sufficient for surfaces of constant curvature.† An exceedingly simple example of such surfaces is furnished by the following *system* of paraboloids,

$$x = e^{\alpha} u$$

$$y = e^{-\alpha} v$$

$$z = a e^{\alpha} u^2 + b e^{-\alpha} v^2,$$

where α is a real parameter. That these surfaces are not applicable follows at once from the fact that the square of the element of arc,

$$ds^2 = e^{2\alpha}(1 + 4a^2u^2)du^2 + 8ab\,uv\,du\,dv + e^{-2\alpha}(1 + 4b^2v^2)dv^2,$$

is not independent of α. The Gaussian curvature is, however, the same at corresponding points of all the surfaces of the system since it is given by

$$K = \frac{4ab}{(1 + 4a^2u^2 + 4b^2v^2)^2},$$

an expression independent of α.

In a recent note‡ a far more general system of surfaces possessing these properties was considered but attention was not explicitly directed to this point.

YALE UNIVERSITY,
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* Two notes, Zur Theorie des Gauss'chen Krümmungsmaasses, *Leipziger Berichte*, vol. 45 (1893), pp. 163 and 170 respectively. See also Scheffers, *Anwendung der Differential- und Integralrechnung auf Geometrie*, vol. 2, p. 276.

† Bianchi, *Vorlesungen über Differentialgeometrie*, p. 186.

‡ ANNALS OF MATHEMATICS, vol. 4 (1902–03), p. 107; in particular, see the theorems on p. 113.